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Essay

What Can Be Said about the Number 13 beyond the Fact that It Is a Prime Number? *

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The story of the number 13 that goes back to ancient Egypt is told. The mathematical significance of 13 is briefly reviewed and 13 is discussed in various contexts, with special reference to the belief of many that this number is a rather unlucky number. Contrary examples are also presented. Regarding everything, the number 13 appears to be a number that leaves no one indifferent.

INTRODUCTION

»Numero pondere et mensura Deus omnia condidit.«
Isaac Newton in 1722
(Rozsondai and Rozsondai)[1]

The concept of number is one of the oldest and most useful concepts in the history of human race.[2–5] People have been aware of the number concept from the very beginnings of humankind. However, there was (and there still is) a difference in the use and appearance of numbers in different contexts, some numbers being used more often than others.[6] There is a variety of reasons for the frequent utilization of certain numbers in cultural contexts such as:

– religious [3 in the Christian faith, 5 in Islam];
– dramatic [3 and 9 by William Shakespeare (1564–1616) in *Macbeth*];
– literary [3 and 10 by Graham Greene (1904–1991) in the titles of his successful short novels *The Third Man* and *The Tenth Man*; 5 and 9 by Dorothy Leigh Sayers (1893–1957) in the titles of her novels *The Five Red Herrings* and *The Nine Taylors*. (It is interesting to note that the novel *The Five Red Herrings* was first issued in the USA under a different title – *Suspicious Characters*, but the second edition appeared under the title *Five Red Herrings*.) 5 by Paulo Coelho in the title of his bestselling novel *The Fifth Mountain* and by an-

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other bestselling author, Joanne Harris, in the title of her novel *Five Quarters of the Orange*. 3 by Dan Brown as a pass-key in his novel on code-breaking *Digital Fortress*];

- alchemistic [4 by Robert Fludd (1574–1637), a notable alchemist and opponent of Johannes Kepler (1571–1639), who wrote a beautiful essay on the manifold qualities of the number 4 (see Jung and Pauli[7])];
- symbolic [different numbers depending on the historical, political, cultural or religious circumstances, *e.g.*, 13 was the fundamental holy number in the religion of the Aztecs];
- sporting [5 in basketball, 6 in volleyball, 9 in baseball, 11 in soccer];
- counting systems [7 days in a week, the Aztec week had 13 days, 12 months in a year, 52 weeks in a year that can be divided into 4 quarters of 13 each];
- in betting [one of the most popular chance systems in Italy is Totocalcio, a betting system where the winner has to guess correctly the outcome of 13 soccer matches among three choices: win, draw and defeat – the chance to win is $1/3^{13}$]; *etc*.

In our computer-based world, two numbers are prominent: 0 and 1. Articles and books have been written about these two fascinating numbers, *e.g.*, Ifrah,[8] Pogliani *et al.*,[9–11] Seife,[12] Barrow.[13] Even Charles Dickens (1812–1870) wrote about one in his second novel *Oliver Twist* (London 1839): »Some conjurers say that number three is the magic number, and some say number seven. It's neither, my friend, neither. It's number one.« These words are spoken by Fagin and are followed by a whole page of explanation why this number is so important. The number 5 was also exhaustively treated in a rather specific way, *e.g.* Hargittai,[14,15] Trinajstić,[16] Tolić and Trinajstić[17] and so were occasionally some other numbers.[18] However, the number 13 has not been the subject of either an article or a book. There are a number of reasons for this neglect. Many of them will be pointed out in the text that follows. Our essay aims to reveal the beauty and the magic of the number 13.

ONE OF US (NT) AND THE NUMBER 13

»What's in a name?«
(William Shakespeare, *Romeo and Juliet*)

The number 13 is especially important to one of us (NT), because it appears as the root word in the family name Trinajstić: 13-ić, 13 = trinajst in the Čakavian dialect of the Croatian language; the Croatian language has 3 dialects: Štokavian, Kajkavian and Čakavian.[19] The family name Trinajstić is a rather old Croatian family name (it can be traced to the 12th century in the place called Trinajstići, located above Rijeka) and belongs to the class of Croatian family names with numbers in their roots.[20] Šimunović,[20] an authority on the origin of Croatian family names, hypothesized that the family names reflecting a certain number were motivated in old patriarchal families by the order of birth. Thus, the thirteenth born child got the nickname Trinajstić (the thirteenth, 13 and ić meant the smallest), which later became the family name of the descendants of this child and has remained so until today. Šimunović also mentioned very briefly the possibility that the family name Trinajstić originated from the folk belief in the magic of the number 13. Anyway, whatever the origin of the family name Trinajstić, the number 13 is a lucky number for the Trinajstić clan, which is now dispersed over all five continents although in some cases the name has undergone changes, *e.g.*, a part of the clan in California is now called Trinast (presumably an immigration clerk on Ellis Island had trouble spelling the name Trinajstić and consequently shortened it to Trinast).

Professor Sven J. Cyvin (Trondheim, Norway) liked so much the idea of using the alphanumeric family name that he and his co-workers dedicated a paper to NT, using 13-ić as NT's family name,[21] on the occasion of his appointment to the position of Editor-in-Chief of *Croatica Chemica Acta*, a chemistry journal published by the Croatian Chemical Society since 1927. When Regina, the daughter of NT, produced a dance show, in the printed announcement of her show, she was introduced to the public as Miss Regina 13-ić.

Professor Cyvin also sent a pentagon-shaped letter to NT (because of NT's interest in the number 5 and fivefold symmetry) on October 12, 1989, addressed to N. 13-ić (see Figure 1), which reached NT without any

Figure 1. The photo of a pentagon-shaped letter addressed to N. 13-ić which arrived in Zagreb from Trondheim without any difficulty.

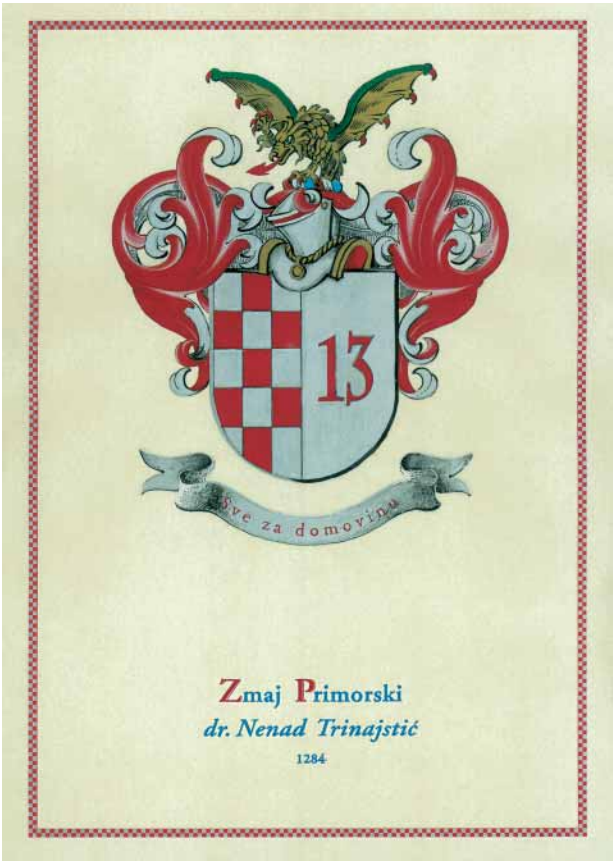

Figure 2. NT's coat of arms containing the number 13. The left side contains a part of the Croatian Coat of Arms. It should also be noted that there are 13 red squares in the Croatian Coat of Arms. The dragon above the NT's coat of arms is an ancient symbol of the *Brethren of the Croatian Dragon*.

problem. Apparently, the European postal system is not prejudiced against pentagonal letters and family names written in the alphanumeric way.

NT belongs to the Croatian fraternal and cultural society called *Brethren of the Croatian Dragon*, which descended from *The Knights Order of the Dragon*, founded in 1408 by the Croatian and Hungarian king Sigismund.[22] NT's coat of arms contains appropriately the number 13 (see Figure 2).

THE NUMBER 13 IN MATHEMATICS

»The number 13 might seem to have no particular mathematical significance, but this is not entirely so.«
(Roger Penrose)[23]

The number 13 is the seventh Fibonacci number, the seventh odd integer, the sixth prime number, the fifth Lucky Number and it is the second prime number in the first twin primes (11 and 13). Its »symmetric dual« number, 31, is also prime and has interesting properties, as we shall see later. The square root of 13 ($\sqrt{13}$) is an irrational already known to the Greeks, who knew that square roots from 2 to 17 (excepting 4, 9 and 16) are irrationals. 13 belongs to the class of prime numbers that are one more than a multiple of four.

As already stated, the number 13 is the seventh member of the *Fibonacci series*:[24] 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233,..., $F_n(= F_{n-1} + F_{n-2})$, where F_n is the n-th Fibonacci number. The Fibonacci series was named after the Italian mathematician Fibonacci (c.1175–c.1250), also known as Leonardo da (of) Pisa or Leonardo Pisano. The use of the subriquet Fibonacci, a contraction of filius Bonacci, son of Bonaccio, for Leonardo Pisano was introduced in 1838 by the mathematical historian Guillaume Libri.[9] There is no evidence that Leonardo referred to himself as Fibonacci or was ever called so by his contemporaries. He actually called himself Leonardo Pisano Bigollo. No one has been able to trace the origin of Bigollo in his name, but it has been speculated that he called himself Bigollo (ignorant peasant) to differentiate himself, who never attended university, from the pretentious ignorant academics of those days. Additionally, his father's name was not Bonaccio: His name was Guglielmo. The problem that gave rise to the Fibonacci series is specified by Leonardo Pisano in his book *Liber Abaci* (*The Book of Abacus*, published in 1202), there serving as a model for breeding rabbits. The Fibonacci rabbit population model is perhaps the first study in mathematical biology, many centuries before the emergence of mathematical biology as a part of biological science. Fibonacci numbers occur so frequently in Nature that one is convinced it cannot be accidental.[25–27] For example, Fibonacci numbers are frequently associated with the number of petals in different flowers (*e.g.*, corn marigolds have 13 petals), Fibonacci numbers are also found in the arrangement of leaves, twigs, and stems (*e.g.*, the ratio of leaves to turns in the spiral is called a phyllotactic and in most cases phyllotactic ratios happen to be Fibonacci numbers), Fibonacci numbers have sometimes been called the pine cone numbers because consecutive Fibonacci numbers have a tendency to appear as left and right sided spirals of a pine cone (often 8 spirals to the right and 13 spirals to the left), *etc*. The Fibonacci sequence is also used as a coding system in the bestselling novel *The Da Vinci Code* by Dan Brown (Bantam Press, New York, 2003).

The 13th Fibonacci number is $233 = 8^2 + 13^2$. If the Fibonacci series starts with zero: 0, 1, 1, 2, 3, 5, 8, 13, *etc*., then the 13th Fibonacci number is $(13 - 1)^2 = 144$. This number is the number 100 in the duodecimal number system of counting and is the only square Fibonacci number apart from 1 (and 0).

The Fibonacci series can be used to generate, for example, the *Lucas series* (1, 3, 4, 7, 11, 18,...), so named after Francois Eduard Anatole Lucas (1842–1891), who gave the Fibonacci series its name. Each Lucas number is the sum of two Fibonacci numbers separated by a single

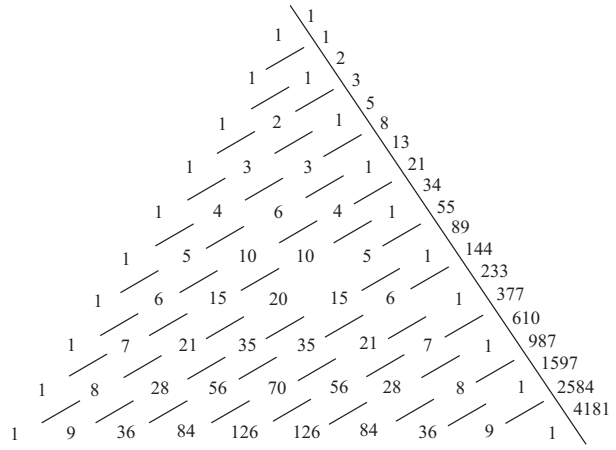

Figure 3. Pascal's triangle and Fibonacci numbers.

Fibonacci number. Thus, for example, Fibonacci numbers 5 and 13 (separated by 8) give the Lucas number 18.

A Fibonacci series is hidden in many patterns that can be expressed in terms of numbers. For example, such a number pattern is *Pascal's triangle* (see Figure 3). One of its hidden properties is that it contains the Fibonacci series.

Although this is called Pascal's triangle because Blaise Pascal (1623–1662) made extensive use of the triangle, it is of very ancient standing, probably more than a thousand years old – it was known to Omar Khayyám (c. 1050–1123) and much earlier to Chinese mathematicians. It is not known whether Pascal had noticed the connection between the triangle and Fibonacci numbers. Pascal's triangle has many interesting hidden properties,[28,29] but these properties are not relevant to the subject of the present essay.

The number 13 is also a number that appears in the *Fibonacci Triangle*.[30] The Fibonacci Triangle is the 2-dimensional array of numbers which are elements of various Fibonacci-like series (see Figure 4).

The number 13 is the fifth Lucky Number. *The Lucky Number series is*:[31] 1, 3, 7, 9, 13, 15, 21, 31, 33, 37, 43,...

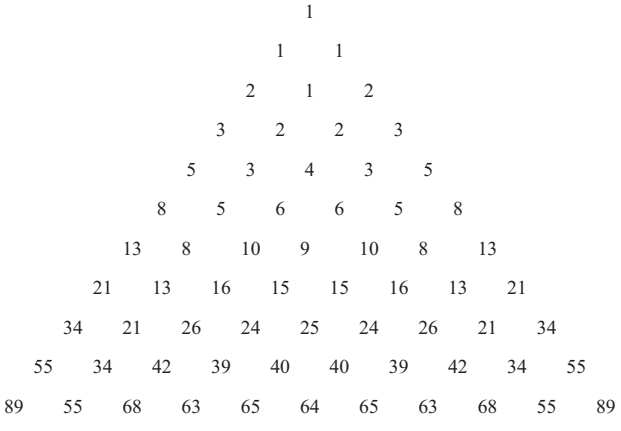

Figure 4. Fibonacci Triangle.

The construction of this series is rather involved: One first writes down the set of integers:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13,
14, 15, 16, 17, 18, 19, 20,...

The first odd number is 1, so one must strike out all even numbers, leaving only odd numbers:

1, 3, 5, 7, 9, 11, 13, 15, 17, 19,...

After 1, the next odd number is 3, so one must strike out every third number:

1, 3, 7, 9, 13, 15, 19,...

The next odd number is 7, one must strike out every seventh number starting with 19, and so on. Numbers that remain are the Lucky Numbers. This procedure is similar to the procedure used to generate the prime numbers smaller than some given number. It was invented by the Greek mathematician Eratosthenes (276–194 B.C.) and is called the sieve of Eratosthenes.[5] No wonder that Lucky Numbers and prime numbers share many properties.

The number 13 represents the sum of the second (3) and fourth (10) triangular numbers, the sum of the second (4) and third (9) square numbers and the sum of the first (1) and third (12) pentagonal numbers. Polygonal (trigonal, tetragonal, pentagonal, *etc*.) numbers represent a series of numbers whose names are derived from an association with patterns shaped like regular polygons. This property of 13, as well as of any other natural number, is the consequence of the set of rules due to Pierre de Fermat (1601–1665) by which he established the relationship between the polygonal numbers and the natural numbers.[28] There are formulas to generate each polygonal sequence. The triangular numbers are given by 1, 3, 6, 10, 15,..., $n(n+1)/2$, the square numbers by 1, 4, 9, 16, 25, …, n^2, the pentagonal numbers by 1, 5, 12, 22, 35,..., $n(3n - 1)/2$, and so on.

There are other interesting regularities involving the number 13. For example, $13 = 2^3 + 5$, where 5 is just the sum of the two preceding prime numbers 2 and 3. The ratio 1/13 equals 0.07692307692307....., where 769230 is a periodic unit. The sum of the first two members of this periodic unit is always 13 (and the sum of the next two is 11).

There are 13 exceptional semiregular polyhedra, called the *Archimedean polyhedra*:[32] truncated tetrahedron, truncated cube, truncated octahedron, cuboctahedron, truncated cuboctahedron, rhombicuboctahedron, snub cube, truncated dodecahedron, icosidodecahedron, truncated icosahedron, truncated icosidodecahedron, rhombicosidodecahedron and snub dodecahedron. A semiregular polyhedron has regular polygons as faces, but the constituting polygons are not of the same kind, and each of its verti-

ces is symmetrically equivalent to every other vertex. All 13 Archimedean polyhedra possess 2-fold and 3-fold rotation axes. The Archimedean polyhedra from the truncated cube to snub cube possess 4-fold rotation axes, whilst the last six Archimedean polyhedra possess 5-fold axes of rotation. This number of Archimedean polyhedra has been shown to be resistant, because the Croatian mathematician Bilinski (1909–1998)[33] and later the Russian mathematician Aškinuze[34] thought that they had discovered the 14th Archimedean polyhedron differing from the rhombicuboctahedron only in that its upper part, which consists of 5 squares and 4 equilateral triangles, is rotated by $\pi/4$ radians. However, all the vertices in the Archimedean polyhedra must be point-group equivalent and this requirement eliminates Bilinski's and Aškinuze's polyhedron from the set of the Archimedean polyhedra. The Archimedean polyhedra came again into prominence after the discovery of buckminsterfullerene, a cage molecule consisting of 60 carbon atoms and possessing the structure that can be best modeled by the truncated icosahedron.[35] The name buckminsterfullerene was given in honor of the architect Richard Buckminster Fuller (1895–1983) who developed the art of constructing geodesic buildings. His pavilion in the shape of a geodesic dome was used to house the American exhibition at Expo '67 in Montreal (Canada). If the discoverers of buckminsterfullerene had been familiar with the theory of Archimedean polyhedra, many initial problems regarding the structure of buckminsterfullerene would have been solved much sooner. The 13 duals of Archimedean polyhedra are called *Catalan polyhedra*,[36] presumably in honour of the Belgian mathematician Eugene Charles Catalan (1814–1894) who rediscovered a series of numbers, now named after him the Catalan series. However, the discoverers of this series were Johann Andreas von Segner (1704–1777) and Leonhard Euler (1707–1783), who used this series to count triangulations of polygons.[37] (Triangulation of a polygon is the division of its inside into triangles). Alas, the number 13 is not a Catalan number, but 14 (= 13 + 1) is.

In *The Science Almanac* (1985–1986 Edition, Garden City, N.Y.: Anchor Press/Doubleday, p. 362) edited by Bryan Bunch, the ten most interesting small numbers are listed. The first is 0 (zero) and the last is 39. The sentence accompanying this number is: »This number is the smallest whole number for which mathematicians have found no interesting property.« This number was selected to this intriguing list on the basis of a paradox typical of mathematicians, that is, its interesting feature is that 39 is not a mathematically interesting number. Well, if nothing else, this number is 3 times 13. In addition, like all two-digit numbers ending in 9, it equals the product of its digits plus their sum: $3 \times 9 + (3 + 9) = 39$.

And now let us say a few words about the symmetric dual of 13, *i.e.*, 31. The connection between these two numbers is best seen in the following expressions: $13 = 1 \times 10^1 + 3 \times 10^0$ and 31 just reverses the two exponents as $1 \times 10^0 + 3 \times 10^1$. 31 is one of the only two known numbers (the other is 8191) that can be written in two ways as successive powers,[31] starting from 1: $31 = 5^0 + 5^1 + 5^2 = 2^0 + 2^1 + 2^2 + 2^3 + 2^4$, but it can be also written as $31 = 3^3 + 2^2$. It is the first prime number the decimal period of whose reciprocal is an odd number of the digits of length, 1/31 = 0.03225 80645 16129 03225, and whose digits of the decimal period have the intriguing property: 03225 + 80645 + 16129 = 99999. Furthermore, 31 is the fifth Mersenne number, $M_5 = 31 = 2^5 - 1$, and the third Mersenne prime, leading to the third perfect number, 496 ($[M_5 (M_5 + 1)/2)$.[4,5,31] *Mersenne numbers* are of the form: $2^n - 1 = M_n$, where n is a natural number. In 1644, father Marin Mersenne (1588–1648) asserted that the only values of n not greater than 257 for which $2^n - 1$ is a prime are n = 1, 2, 3, 5, 7, 13, 17, 19, 31, 67, 127, and 257 (here again we have 13 and 31). $M_{13} = 8191$ is a prime and was known to be prime in medieval times, as well as M_{17} and M_{19}. The first five of these primes are 1, 3, 7, 31, and 127. A perfect number is the sum of its divisors, including unity but excluding itself, *i.e*, 6 = 1 + 2 + 3. Euler (1707–1783) proved that all even perfect numbers are of the form $2^{n-1}(2^n - 1) = 2^{n-1} M_n$. For M_5 we have $2^4 \times 31 = 496$, which is the third perfect number (6, 28, 496). It has been discovered in modern times that M_{67} and M_{257} are not primes. Futher, a conjecture was advanced in more recent times that M_n is a prime whenever n is itself a Mersenne prime. But this conjecture holds until the Mersenne prime $M_{13} = 9191$ is reached, as in this case the 2466-digit number M_{8191} is a composite!

Futhermore, there are two pairs of intriguing relations tying together 13 and 31: (i) $13^2 = 196$ and $31^2 = 961$ (these squares are made up of the same digits) and (ii) $13^3 = 2197$ and $31^3 = 29791$ (these cubes are made up of the same digits plus one digit that belongs to the same set of digits).

THE NUMBER 13 IN VARIOUS CONTEXTS

»13. A notoriously unlucky number.«

(David Wells)[31]

Already in antiquity, the number 13 was considered an unlucky number in many cultures. For example, Philip, the king of ancient Macedonia and the father of Alexander of Macedon, was murdered in the theatre a few days after adding his statue to the 12 statues of gods. Even today there are some people who suffer from triskaidekaphobia, an irrational fear of the number 13. The origin of the present-day fear of the number 13, especially among Christians, can be traced to the 13 (Jesus Christ and 12 apostles) who sat at the table at the Last Supper. Note that as soon as Jesus was murdered, Judas also disappeared (committing suicide) restoring, thus, the prime number of apostles, *i.e*, 11. The symbol known as *crux gemmata*

– a cross bearing 13 gems – is the Christian ideogram for Christ and his 12 apostles. Faced with the strange preference for some numbers in many religions, we can really wonder how far and deep the Pythagorean number mysticism spread over the ancient world and if religions worshipped numbers more than God. Additionally, the 13th chapter of *The Revelation of Saint John the Divine* (*The New Testament* – King James version) talks about the scary Beast who is blaspheming against God.

The number 13 plays also a role in the Egyptian mythology.[38,39] The envious God Seth (a personification of Evil) killed and hacked up the body of his brother, the God Osiris, into 14 pieces, 13 bits of flesh, bone and sinew and he scattered them over the Nile. In the chapter *Twenty-one Women* of the book *The Egyptian Book of Dead*, the reader is told of a man who walks in the House of Osiris along some kind of purification path and at each pylon meets a temptress. The 13th woman is the Goddess Isis, the wife of Osiris, who represents death and eternity and she is the only blessed lady and the only woman that shares divine attributes.

Friday, the 13th is considered by many to be especially unlucky day.[40] Thus, many people do not start anything on this particular day. However, the followers of Saint Anthony of Padua consider Friday, the 13th as a lucky day for a very strange reason: Saint Anthony (Lisbon, 1194 – Padua, 1231), who is a very popular saint, died on Friday, the 13th. However, Friday, the 13th was fatal for the Templars, the knights of a religious order established as the Order of the Poor Knights of Christ and the Temple of Solomon in 1119 in Jerusalem for the protection of pilgrims and the Holy Sepulcher. They soon became very influential and spread all over Europe. They appeared in Croatia in the second half of the 12th century. Pope Clement V (pope from 1305 to 1314, who transferred the Holy See from Rome to Avignon) decided to curb their independence, expansion and influence. On Friday, October 13th, 1307 under the orders of Pope Clement V and in concert with France's Phillipe IV (king from 1285 to 1314) hundreds of Templars were killed and the Order was soon abolished (1312).

The ill-omen of the number 13 might have been realized in some cases, *e.g.*, in the war between the Confederacy and the Union, and Iraq's aggression against Kuwait: There were 13 states in the Confederacy and Kuwait was considered by Iraq to be its 13th province. Maritime law used to require a medical doctor on a ship if there were 13 passengers; hence the number of passengers to avoid the ill-omen had be 12 or less, or 14 or more. Note that smaller dinner parties usually had 12 or 14 people at the table. What happens when there are 13 people at a dinner party was used by Agatha Christie (1891–1976) in her novel *13 at Dinner*. We should also mention that the astronauts dislike the number 13 as well, especially after the disaster of Apollo 13. It is nothing new if we say that some people from all walks of life are superstitious, especially sportsmen such as baseball players, tennis players, soccer players (refusing to wear a uniform with the number 13), but also airplane pilots, travellers (many people avoid travel by plane on Friday, the 13th), even scientists.

The superstition of the famous Austrian composer Arnold Schönberg (1874–1951) was extreme: he spelled the name of Moses' brother Aaron as Aron in the title of his opera *Moses und Aron* so that it would have 12, rather than an unlucky 13 characters. Perhaps this fear of the number 13 was justified in his case, because Schönberg never succeeded in completing *Moses und Aron*, although he started to work on this opera in 1927 and had completed two acts by 1932. He died on Friday, July 13, 1951. It may be his fear of the number 13 was the reason why Schönberg initiated dodecaphony – a twelve-tone method of composing?

In the same vein is the story of how the great comedian Stan Laurel of the Laurel and Hardy movies got his name.[41] He was born as Arthur Stanley Jefferson. When he first started in show business, he used the name Stanley Jefferson and then he shortened it to Stan Jefferson. His beginnings in show business were rather unsuccessful and one day, quite by accident, he counted the letters in his name – there were 13 letters in Stan Jefferson. To avoid superstition connected with the number 13, he picked up, again quite accidentally, Laurel as his professional name, and when he became known in pictures, he legalized this name. His career greatly improved with the change of the name, and he eventually became, together with Oliver Norvell Hardy (known privately as Babe Hardy and in movies as Ollie Hardy), one of the greatest movie stars of the first half of the 20th century.

As a result of the modern-day superstition related to the number 13, many airplanes do not have the thirteenth row, and many hotels have no rooms and some have no floors bearing the number 13. Thus, Arthur Hailey writes in his bestselling novel *Hotel* (Bantam Books, New York, 1967, 10th printing, p. 80): »Like all sizeable hotels, the St. Gregory pretended not to have a thirteenth floor, naming it the fourteenth instead.« On the other hand, John Michael Coetzee (The Nobel Prize for Literature 2003) writes in his novel *Elizabeth Costello* (Secker and Warburg, London, 2003, p. 24): »His (room number) is 1220. Hers is 1307. He thought that floors went twelve-fourteen, that that was the rule in the hotel world.« Perhaps some hotel managements do not believe that the number 13 is an unlucky number, but their guests might.

There is a paradox regarding the number 13 superstition. This number seems to be universally present in mammalian microtubules.[42] A microtubule is a hollow protein tube, normally consisting of 13 columns of tubulin dimers. Every tubulin dimer is a globular protein pair consisting of two parts, called α-tubulin and β-tubulin, each

composed of about 450 amino acids. One of the numerous functions of microtubules is that they serve as tracks for the transport of cell organelles in nerve cells. Moreover, it has been suggested that microtubules control the activity of the brain.[23] Therefore, we have a rather paradoxical situation: In the mind of the person fearing the number 13, the fear is created by processes at the roots of which lies the number 13.

On the other hand, there are some people (*e.g.*, NT) who are uncommonly fond of the number 13, suffering from *triskaidekaphilia*, an unreasonable liking for the number 13. It is nothing unusual that some people have a preference for a given number. For example, the popular American writer Charles Bukowski (1920–1994) stated in his last novel *Pulp*, published in 1994 (Black Sparrow Press, Santa Rosa, CA, 13th printing, p. 155): »But most numbers worried me. I only liked 3, 7 and 8 or combinations thereof.« Dudley Herschbach, who shared the Nobel Prize for chemistry in 1986, likes the number 17. Another Nobel Prize winner John Van Vleck (shared the prize for physics in 1977) also invariably used the number 17 whenever he had the occasion to refer to an arbitrary number. In Italy, the number 17 is considered an unlucky number because the Roman letters for 17 are XVII. This can be taken as an anagram of VIXI, which means I lived implying I died. Like 13, also 17 has a symmetric prime companion 71. For example, 11, being itself symmetric, has itself as a companion and the symmetric companion of 19 is not a prime number. It should be interesting to know if the symmetric prime companions of the prime numbers are also infinite in number.

In ancient Greece, the 13th member of a group was always the most powerful. Thus, in the Greek mythology Zeus, who is escorted by 12 gods, is the most powerful god and ruler over all the others. In some other ancient cultures, like that of the Vikings, the number 13 was the sign of a good omen. Thus, this theme was used in the bestselling novel *Eaters of the Dead* by Michael Chrichton. The following sentences come from that novel: »The party of Buliwyf must be thirteen, and of these one must be no Northman, and so you shall be the thirteenth... This, I was told, was to draw the attention of Odin, one of the number of their gods, so that this Odin would look with favor upon the journey of Buliwyf and twelve men. Also, this I learned: that the number thirteen is significant to the Norsemen, because the moon grows and dies thirteen times in the passage of one year, by their reckoning. For this reason, all important accountings must include the number thirteen. Thus Herger said to me that the number of dwellings in Trelburg was thirteen and also three more, instead of sixteen, as I have expressed.« When this novel was reissued, the title has been changed into one much more appropriate *The Thirteenth Warrior.*

There is also a book by Arthur Koestler (1905–1983) entitled *The Thirteenth Tribe* about the mysterious Khazar Empire and its heritage. The Khazars were people of Turkish origin. Their empire, called Khazaria, at the peak of its power (from 7th to 10th century), extending from the Black Sea to the Caspian Sea (also called the Sea of the Khazars) and from the Caucasus to the river Volga, was positioned between the Eastern Roman Empire and the followers of Mohammed. The Khazars chose a surprising way to resist the pressure from both Christianity and Islam – they converted to Judaism – hence the title of the book *The Thirteenth Tribe* – 12 biblical Jewish tribes and the Khazars (the title of the book was suggested by the editor, Harold Harris). It could be that many Jewish communities all around the world are of Khazar origin, *e.g.*, of non-semititic origin! *The Thirteenth Warrior* and *The Thirteenth Tribe* were published in the same year (1976) and are based on the writings of an Arab traveler named Ahmad ibn-Fadlan about his trip to the King of Volga Bulgars as ambassador of the Caliph of Bagdad from June 21, 921 to May 12, 922 when he reached his destination. The difference between the books is in that Crichton spells the Khazars as Hazars and his novel is centered on the Vikings and in a greater part on the legend of Beowulf, while Koestler uses the writings of ibn-Fadlan, since this text is the most important source of information on the Khazars, to support his somewhat speculative, but convincing, story about the history, culture and migration of the Khazars.

There is a film under the same title based on the novel *The Thirteenth Warrior.* This film directed by John McTiernan with Antonio Banderas and Vladimir Kulich was made in the USA in 1999. There are also other films with 13 in the title, such as *Thirteen Hours by Air* (made in 1936). It is quite appropriately about a transcontinental hijacking. However, the first film (comedy?) Alfred Hitchcock (1899–1980) attempted to make at the tender age of 23 (in 1922) had the working title *Number Thirteen* and was never completed.[43] Hitchcock was not much luckier with another number movie, a quickie routine thriller called *Number Seventeen*, made in 1932 in England. However, his English movie with 39 (3 times 13) in the title, *The Thirty-Nine Steps* (based on John Buchan's (1875–1940) novel of the same name), made in 1935, was phenomenally successful with the public and the critics. Some numbers were unlucky and some lucky for Hitchcock. It is not unusual for movies to have numbers in the title. Here we recall the 1952 successful movie *Five Fingers* (director Joseph L. Mankiewicz) with James Mason and Danielle Darrieux about the famous Second World War spy under the code name Cicero, which was retitled when shown in Croatia to *Operation Cicero*, since it was believed that the original title would not attract Croatian viewers.

In the USA, during the long-lost time when there were friendly bread shops along the streets, a baker's dozen meant 13 items. There were 13 initial colonies that form-

ed the USA. These are symbolized by 13 stripes (7 red stripes and 6 white stripes) on the American flag, and originally it had 13 stars, one for each state, though there are now 50 stars symbolizing the 50 states of the USA. There were 13 signers of the Declaration of Independence. The zoological name of chipmunk is the 13-lined ground squirrel. The 13th year of life is the youngest teenager age. The Hawaiian language uses only 13 letters: A, E, I, O, U, H, K, L, M, N, P, W and one letter with a diacritical mark.

There is a sonnet by the New Zealand poet Harry Johnson using 13 (it was published on December 5, 1999 in the weekly Hrvatsko slovo (Croatian Letter) with a selection of poems of another 9 New Zealand poets):

Sonet XCIV

thirteen nonrhymed nonmetric lines
thirteen nonrhymed nonmetric lines
thirteen nonrhymed nonmetric lines
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thirteen nonrhymed nonmetric lines

The structure of this sonnet is rather unusual: it starts with a stanza containing 8 lines and then each following stanza contains half the number of lines of the preceding stanza, ending with a single-line stanza. The sonnet is a verse form of Italian origin and in the standard form, it consists of 14 lines usually structured so that the first two stanzas possess 4 lines (two quadrinas) and the last two 3 lines (two terzinas).

The number 13 also appears in the art of painting, *e.g.*, Eduard Manet (1832–1883), who initiated French impressionism, made 13 portraits of Berthe Morisot (1841–1895), another French impressionist.

In Denmark, the student grades in schools and universities are 0, 3, 5, 6, 7, 8, 9, 10, 11 and 13. The grade 13 is given only to impeccable students!

The 13th element in the Periodic System is Al, which for a long time, before the introduction of electroanalytical separation methods, was so difficult to obtain that its price competed with the prices of gold and silver. Reversing 13 into 31, the corresponding 31th element Ga belongs to the same group as Al – it is just underneath the position of Al – this element is liquid at room temperature. Such an atomic-number reversal for successive family members is only shared by the pair Br (the 35th element) – also liquid at room temperature – and I (the 53rd element). C-13 is the *ca.* 1% carbon isotope with spin ½, which is of paramount importance in NMR; C-13 NMR is a well-developed branch of spectroscopy and together with H NMR is one of the basic tools for the structure elucidation of organic compounds. Similarly, P-31 NMR is also a well-developed branch of NMR. P-31 has spin ½, and is the most abundant isotope of phosphorus and P-31 NMR is very important in biochemical and biophysical studies.

There is an informal chemistry journal entitled *CHEM 13 news*, published by the Department of Chemistry, University of Waterloo, Ontario N2L 3G1, Canada, which is aimed at chemistry teachers in high schools and junior colleges. Its web address is: htpp://www.science.uwaterloo.ca/chem13news/ The following explains the origin of the number 13 in its title. *CHEM 13 news* was started in 1968 at a time when the high school program in the Province of Ontario ended with Grade 13; hence 13 in the title. Grade 13 disappeared in 1990, but the editors have become attached to 13 and have kept this number in the title.

CONCLUDING REMARKS

»As Enrico Fermi once said, one should never underestimate the pleasure we feel from hearing something we already know.«

(Steven Weinberg)[44]

Some mathematical properties of the number 13 are presented. One so far undiscussed property is related to the concept of geminal prime numbers. For example, in the second decade of natural numbers there are two couples of such numbers: 11–13 and 17–19, symmetrically disposed. It seems that geminal prime numbers are infinite, though this has not been proven. Is there an infinite or finite number of couples of geminal prime numbers symmetrically placed in a decade like the aforementioned pairs of prime numbers 11–13 and 17–19?

The numerical superstition concerning the number 13 is discussed. People may be indifferent to 0 or 1, may be aware of 5 by virtue of having five fingers on each hand (though there are some people with six fingers, even on both hands, whence then they presumably are (sub)consciously aware of the number 6), but very few, if any, are indifferent to 13 when dealing with this number and especially so when it is the 13th day of a month and even more if that day is Friday. Though the majority of people think this number is unlucky, contrary cases have been noted. The appearance of 13 in several fields of human

endeavor has also been outlined. Considering all said, the number 13 is not just another number, it is a number with many virtues!

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»A number is the number of something.«
(Zvonimir Šikić)[45]

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SAŽETAK

Što se može kazati o broju 13 osim da je prim-broj?

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Dana je kratka povijest broja 13, koja počinje u drevnome Egiptu. Ukratko je prikazan broj 13 u matematici te su navedene zanimljivosti o broju 13 iz mitologije, kršćanstva, povijesti, književnosti, umjetnosti, glazbe, filma i kemije. Spomenuto je vjerovanje da je 13 nesretan broj, ali i suprotno vjerovanje.